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101

Quizzes

*to prepare yourself for interview*

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*to prepare yourself for interview*

for programmers and economists

**New York**  
**1999**

**Olga Galina, Andrey Itkin, Yakov Pesin. 101 quizzes for preparing yourself to pass an interview (for programmers and economists). World Scientific, 1999.**

**ISBN**

Authors of this book are well-known mathematicians and programmers. They live and work in USA therefore are well familiar with a problem how to prepare yourself to pass an interview. 101 quizzes are collected within the book that either were ever suggested for programmers and economists at real interviews in US firms and companies or are close to that in spirit. To solve the most proposed problems reader does not need to have a special mathematical education, it is supposed that a basic course of an elementary school is sufficient in doing so. A detailed solution of each problem is presented. The book might serve as a good training aid for people looking for a job in programming or economics however one may also treat it simply as a collection of quizzes on popular mathematics. The book also contains some useful information for programmers and economists of the former SU seeking a job in USA, for instance a scheme of a typical interview, questions often asked at interviews, description of some problems of the visa's supply etc. References to Internet pages that contain information about job opportunities in USA and publish some other useful advises for job seekers are also given.

This is a first book of such a kind where authors made an attempt to offer an algorithm how to prepare yourself for interview from the standpoint of mathematics, logic and just a common sense.

For programmers, economists, mathematicians, physicians and simply for all lovers of the amusing mathematics.

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**ISBN**

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## Acknowledgments

This book could not be issued without a kind help of our friends and colleagues. Being aware of author's bias towards solving and collecting various mathematical and logical puzzles, they permanently provided us by new exemplars of such problems. But since the time we made a decision to gather them within one book we just started purposefully persecute all people whom we were acquainted with forcing them to bring us more and more new and newest problems that had been offered at interviews in various firms and companies. Fortunately, by that time we ourselves and basically our colleagues managed to get playing as the role of interviewer so an opposite role being interviewed when looking for a job. That is why they had what to impart to us. Each of them made his own and significant contribution to our mutual work. Therefore we take a chance to express our undying gratitude for their participation and would like to name them personally. Maxim Shclover, Ilya Olshanetskii, Efrem Spravtsev, Vladimir Melomedov, Alex Kononenko, Sergei Verbitskii – our great thanks to all of you.

Some of problems presented in this book were not suggested at real interviews (or at least we are not aware of that fact if it took place) but are close in common spirit and sense to the problems that interviewers use to nonplus probationers. We found them in the books on popular mathematics and programming, in Internet or also heard about them from our friends and colleagues. We considered it as our pleasant duty to attach this book by a list of references to these sources. Thank to all our predecessors who have famously worked for popularization of mathematical methods of programming and economics.

Despite this is not the first book of the authors (but really so far each of us wrote and published books separately) but our first attempt to realize ourselves in another area – an area of popular science. Certainly, pessimist will argue that passing an interview is mostly a kind of art, not science, and perhaps this is true. However, we considered it to be quite reasonable to use scientific approaches when preparing yourself for interview. No doubt that just our editors prevented us to write this book in a usual scientific manner using many formulae and a specific scientific language, and thus helped to make reading easier. Their advises considerably improved the final version of the book and we sincerely thank them for their professional and impressed work.

Names... ...Thank you!

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## PREFACE

The interview. This word makes hearts of those who have ever gone through an interview beat faster. Those who are about to take an interview for the first time get merely shaken when they hear this word. Even «professionals» who have gone through interviews several times and have gained a great deal of experience feel chill and often refuse to discuss the subject. Why is this word so frightening? An interview may last several hours and the interviewee may be asked to solve a number of tricky problems. For some people an interview may become somewhat like a nightmare: stressful days and sleepless nights preceding the interview, «silly» questions addressed to friends like «Can I put one of my legs onto another one while sitting in front of the interviewer?» and so on and so forth. Of course, smart and vigorous people (and they constitute the majority of those who are invited for interviews) raise natural question—How can I avoid this nightmare? Is there any efficient way I can use to prepare for an interview? Somewhat like a collection of «trustful» instructions, a «manual» where I can find answers to most of the «silly» questions as well as solutions of «typical interview problems».

The authors have also come across these questions. Dear reader, the result of this idea is now in your hands.

As you, our reader, already understood the idea was to collect in one place «typical interview problems» and other related «stuff», which can be used for preparing to an interview. Of course, we decided to restrict ourselves to the areas of our own expertise: mathematics, computer science, mathematical methods in economy and a bit of physics. We know almost nothing about interviews, which for example, ornithologists have to go through. Should they show skills to run fast in a dense forest or ability to fly (how else can one get information on wild birds? You can see how much we know about ornithology that is of course, an interesting and useful discipline). We do not want to give any incompetent advises and therefore, ask specialists in other fields who may also look at the book to excuse us.

Following the wise words that «every new idea is just a well-forgotten old one» we decided to look carefully through the existing literature on the subject. We found a number of very good books where some ways for preparing for interviews are considered as well as some quite reasonable advises are given on how to write the resume. We also found some interesting Web sites where various aspects of the interview are discussed. There some people with successful careers share some useful thoughts about interviews and «how to fight them». However, we were not able to find any reasonable collection of problems, which were given to interviewees who were looking for positions in mathematics, computer science business, economy, etc. Although we found couple sites with interesting collections of mathematical, logical, and other «brain-breaking» problems but they were not designed as «typical interview problems» and many of them were much above that level.

It is our understanding that «interview problems» are quite specific and their level and type depend pretty much on the personality of the interviewers. Indeed, they probably are not so much interested in knowing how good you are in solving some tricky problems (we bet you would never face any such problem in your future work). Ultimately, they want to see skills in logical thinking, your ability for taking non-standard decisions, etc. And they can use various ways to try to figure this out. Much also depends on interviewer's personal experience, taste, and preferences. Whether he or she uses problems which were carefully prepared or likes to improvise during the interview (perhaps, depending on the interviewee answers). Therefore, it seems very difficult if not impossible to completely formalize the preparation for the interview. However, in our opinion one can try to work out some skills, which could be very useful in solving «typical interview problems». Most important these skills may help you in your future work and thus in passing the interview. We sincerely believe this can be achieved by thorough working through «typical interview problems».

Where can one find these «typical interview problems»? There are many books on popular mathematics and computer science which contain a lot of puzzles (see references in the book). Among them are remarkable books by Perelman that helped get into mathematics many generations of Russian students. They are written in Russian

and unfortunately are not translated into English. (By the way we know some of the interviewers who often offer some problems taken from Perelman's books). We have collected big number of «real interview problems» which otherwise were scattered throughout different books. We also added other problems which were offered on interviews and which were not published anywhere (and thus went «from mouth to mouth»).

We divided all problems into groups: logical problems, finding strategy, geometrical, and physical problems. This division is somewhat arbitrary since solutions of many problems require both logical thinking and some level of mathematics and even physics (though usually quite elementary). We also dared to classify our problems according to the level of their difficulty. So, we divided the problems into five groups marking by (\*) those which are most simple and by (\*\*\*\*\*) those which are most difficult. One can immediately draw an analogy with classification of hotels or of cognac maturity. Well, it would be not bad if after solving a problem of the level (\*\*\*\*\*) you would feel yourself during the interview as comfortable as if you were in a suit of a five-star hotel or as pleased as after a glass of a five-star cognac.

This book should not be considered as a textbook in mathematics or computer science. In order to solve most of the problems the reader needs to know some basic mathematical concepts on the level of a high school math course as well as some core notions on programming using the language C. Few problems which require higher level of mathematics are marked by (o). For reader's convenience in section «Solutions» we provide some core mathematical facts needed to solve a given problem.

Finally we divided our problems on «real interview problems» i.e., those that were given on some interviews and «problems that might have been given at an interview», i.e., those that are close to «real ones» in level and spirit. In our opinion the latter are very useful in preparing for an interview. Each problem in the book is presented by three personages. The «Stater» (abbreviation S) is the one who states the problem formally. The «Arguer» (abbreviation A) is the one who argues conditions and requirements in the problem and sometimes provides an involuntary hint for a solution. The «Tester» (abbreviation T) is the one who tests the level of the problem, marks it, and provides a comparison with other problems. We hope that this SAT-team (do you recognize the famous test?) will «animate» the problem and make it more attractive. If this Preface did not completely evaporate your interest, if you still want to evaluate yourself by trying various puzzles then go ahead and turn to the next page.



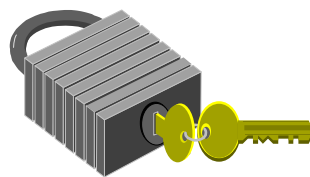
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# Quizzes of real interviews



## 1.1. Common sense



### 1. How to send brilliant\*\*\*\*

S:

One guy wants to pass a brilliant to his friend that lives in another city, by mail. The post service is organized in such a way that any stuff can be delivered only being placed in a still box locked by a padlock. Otherwise all the content of an unlocked box is grabbed. So how does his friend can get an access to this brilliant?

A:

Actually it happens in the contemporary Russia. This guy has to send this brilliant from Moscow to St.Petersburg. The post service works very bad. Not too bad, but this service is quite unreliable. But at least locked boxes could be delivered (it means that nevertheless something is good in the contemporary Russia!). However it rises a problem: how does this friend in St.Petersburg will open the box? Indeed, he has no key. There are some ways to pass the key, but if, for instance, you place the key in the locker it will be grabbed. If you put this key in another box and send it to St.Petersburg you must lock it otherwise the key will be grabbed. But now how to open the second box with the key inside? People in Russia use very ordinary padlocks, but nobody can produce a key having only a locker. So does nobody can do anything. Suppose trains do not go from Moscow to St.Petersburg, aircrafts do not fly, so there is no any opportunity to pass the key from Moscow to St.Petersburg except by using mail. Thus, it is unclear how the guy's friend may get an access to this brilliant (i.e. he wants to have it in hands, definitely not in box).

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T:

Despite this problem is highly subtle it has a “fair” solution. Needs solely a logical mind and a quick wits. I have suggested it for many people but only few of them managed to solve it. Surely give it 4 stars.

2. Two robs\*\*



S:

Given two ropes that burn non-uniformly. Each rope being set on the fire from one side completely burns down during an hour. How one could measure 45 minutes using 2 such ropes?

A:

The essence of the problem is that each rope burns non-uniformly. Otherwise I would fold it in two, than again, note where is three fourths and then set it on the fire just at this point. But as it is it is unclear what to do.

T:

This is also a problem to check your quick-wittedness and whether you have a non standard mind. Something about 2 stars.

3. Alphabetical series\*

S:

Given the series *O T T F F S S E N* ..... Could you continue it?

A:

Sorry, I have no comments. It is a perfect statement of the problem. But once I have got this problem by myself at a real interview. Fortunately, I solved it.

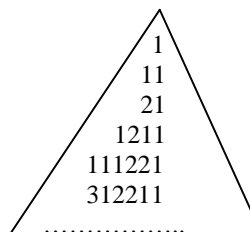
T:

This problem is not so complicated but funny. My evaluation - 1 star.

4. Numerical series\*

S:

Given the triangle



What should be the next row?

A:

Despite I knew this problem for a long time, once in Internet I have found a note that the sum of digits in each row is a Fibonacci number. Thus, to avoid any ambiguity it is necessary to give the next row in the triangle.



T:

According to my observations women solve this problem much faster than men. Apparently it also requires some kind of non-standard logic which perhaps is close to “women” logic. This problem is as not so complicated as funny. My evaluation - 1 star.

5. Problem of crossing\*

S:

Two people come to the bank of the river and see an empty boat. But it may carry only one person. So how they both can cross the river?



A:

This boat with two people on the board sinks, it has no enough floatation. Unfortunately, these guys do not know how to swim. Even if one of them holds the boat he can not swim. And the water is practically freeze, Uh-h..! Thus it is better keeping dry. It would be good to find at the bank something that can float, for instance some log, but there is nothing. So yes, indeed how these guys can cross this river? Unfortunately, for the time being we can not fly either.

T:

Again it is a problem of quick-wittedness but with a quite different approach for solving it. Despite it is rather intricate but not too hard. Therefore it does not deserve more than a star.

## 1.2. Logic problems

6. Town of liars\*\*

S:

There are two towns: the first one (T) where all citizens always tell truth and the other one (L) where all people always lie. You stay at a fork of two roads. One of them goes to town T while the other one – to town L but you are not aware of to what town which road goes. Fortunately, you meet a citizen of one of these towns but unfortunately you do not know where he lives. On your question he answers only “Yes” or “No”. How could you find a right way to town T if you might ask him only one question?

A:

It is clear that the usual question like “Does this road go to the town of liar?” is not suitable in this case. Indeed, let us assume you point out to a certain town which actually is town L. If you met the liar he/she answers “No” in such a case while the truthful person say “Yes”. So you have no idea what this town is. Therefore, your approach has to be more sophisticated.



T:

Yes, but not so much. I believe the right idea deserves two stars.

### 7. Four sages\*\*

S:

Four sages contended with one another who of them is the wisest. An old man decided to help them. He took 5 caps from a bag – 4 black and one white, put black caps up on a head of every sage respectively, but the white cap returned back to the bag. Each sage does not know what color is the cap on his own head but can see what caps are on all the other sages. Who first manages to guess the color of the cap on his head – is the cleverest one. Upon some time one of these sages has determined that he is dressed in a black cap. How did he know it?



A:

The simplest way would be to ask somebody about it. But each sage is in competition with the others so he does not want to help his competitors. The old man also keeps silence. And nobody else is hereabout.

T:

This is a logic problem of another type. Actually, not that difficult. 2 stars.

### 8. Five sages\*\*\*

S:

The same problem as the previous one but now there are 5 sages while the old man has 7 caps – 5 black and 2 white. Again he put all black caps on these sages and returned back to the sack all white caps. And again one of these sages manages to guess what color is the cap on his head. How did he do it?

A:

I have a good solution – he did it in the same way like for the previous problem! But it is a joke, of course.

T:

I definitely recommend resolving the previous problem first because some advance logic has to be applied here. The additional sage – the additional star. And the number of caps has also been increased that increases a possible number of variants of the solution.

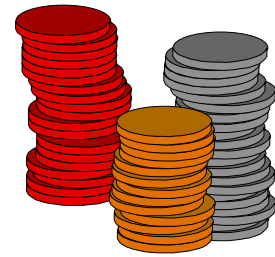
### 9. Lost dollar\*\*

S:

Three men had dinner at a local restaurant. When dinner was over they paid \$30.00 total to the waitress (so each paid \$10.00) and left. Suddenly the waitress realized that she charged them five dollars more. She gave extra five dollars to the boy who assisted her and asked him to run and return the money to the three men. The boy thought it would be fair if he would take two dollars out of five and would return only the remaining three dollars. He did so and hence, each man got one dollar back. Finally, the three men paid  $27.00 = 3 \times 9$  dollars for the dinner and the boy got two dollars. This results in total 29.00 dollars. Where is an extra dollar?

A:

A very actual problem. The moral is - have a calculator if you can not to count your expenses in mind. However, may be they gave this extra dollar as tips? Otherwise it is some trick but I would like to know who is this adroit juggler who pocketed this dollar?



T:

Yes, there is a trick but nothing criminal. Imagine that you are a judge and investigate this matter. You definitely should find it!

### 10. Honest people\*\*

S:

16 people say the following statements one after another:

1: There are no honest people in this room;

2: There is at most one honest person in this room;

3: There are no more than two honest people in this room

.....

.....

16: There are no more than 15 honest people in this room.

How many honest people are in the room?

A:

Let me imagine same problem but with 2 people in the room. The first guy asserts the absence of honest people while the second one says that there is no more than 1 honest person. If the first is a liar than the second tells the truth and we solved the problem. But to analyze the situation if there are so many people is much harder.

T:

That is right. It means you have to find another way which allows you to find the solution under an appropriate number of people in the room. Try it!

### 11. Town of unfaithful wives\*\*\*



S:

N family pairs live in a town. All pairs are divided into two groups: happy pairs where a wife is faithful to her husband, and unhappy ones where respectively she is unfaithful to him. Given that each husband certainly knows about any women in the town except only his own wife whether she is faithful to her husband or not. In this town there is a law: each husband who has learned about his wife's adultery must bring her to the court same day. One day a pilgrim comes to the town and say: " I know that there are adulteress in your town. How many days have to pass after in order t to bring all adulteresses to the court?"

A:

Despite man's solidarity in this case husbands can not exchange information with one another (may be they do not want?). In other words nobody will come to you and tell that your wife is unfaithful to you. And that is right! Who knows how this miserable man will react to this information. The best case is if only his wife will suffer from him, but there is no guarantee that a part of his fury will fall on you.

T:

This problem seems to be not that easy. Here is a hint: try to work on problems about sages first. For this one I would give an additional star.

## 1.3. Search of strategy

### 12. Herrings in a barrel\*\*\*

S:

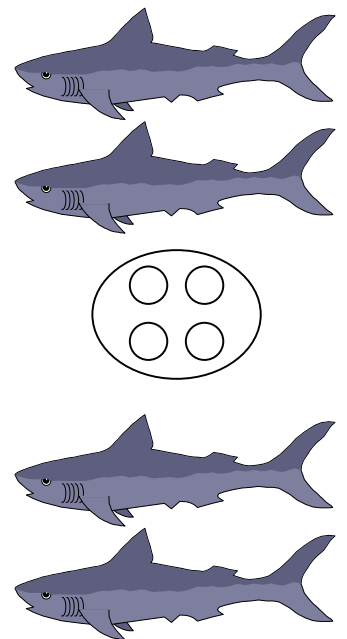
You stay in front of a circle barrel which lid has four symmetrically located holes. In every hole there is a herring in a vertical position. These herrings could stand in the barrel either head up or tail up. Given that the lid is opened when all herrings are oriented in the same directions, i.e. all head up or all tail up. You may do the following action: put simultaneously your hands in any two holes, take 2 herrings off from these holes, look at them and then put them back either having kept their orientation or turned back any of these two herrings (or even both). After that the barrel is rotated and stopped so that you do not know from what holes you did take herrings. Again you may take off two herrings from any two holes etc. What should be your strategy in order to open the barrel? What is the minimum number of steps to achieve this result?

A:

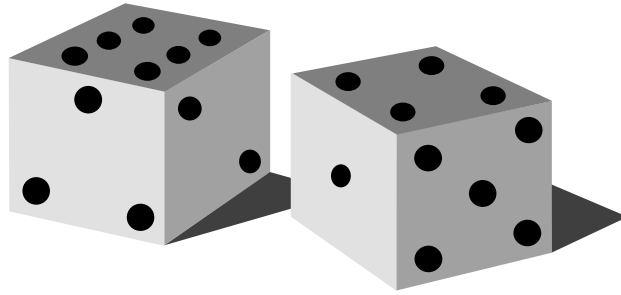
Well, it looks like I could put hands in the holes in two different ways: to adjacent holes or to opposite (diagonal) ones. But even though it may happen that theoretically I could never ever put my hands in some of the holes! This worries me! It must be hard problem!

T:

Oh, do not worry – this problem has a solution. And do not forget that the barrel can be opened in two cases: when all the herrings are standing head up or vice versa – tail up. I thought about two stars first but you convinced me that 3 stars is fair.



### 13. Dices\*\*



S:

You roll the dice and get the certain number  $k$  from 1 to 6. You have a choice: either to take  $k$  dollars and stop the game or to roll the dice again and take another amount of dollars equivalent to the new obtained number. Let us assume that you have only 3 attempts. What should be your efficient strategy to get maximum money?

A:

In case of two possible attempts your strategy could be as follows. If after the first throw you have got 1, 2 or 3 points, the probability to get more within the second attempt exceeds 50%. Thus, it makes sense to roll the dice again. And vice versa if you have got 4, 5 or 6 it makes sense to take your money off. But what do you have to do with 3 attempts?

T:

The probability theory is involved here. This may scare the reader who is not familiar with it. But do not panic, this problem can be solved in many ways, and in particular even just applying the common sense. That is why – 2 stars.

### 14. Choice of balls\*\*

S:

Given 3 closed boxes with a ball inside each one. Two of these balls are black while the last one is white. You are asked to choose the box where according to your opinion is a white ball. Then one of two other boxes is opened and so you can see that there is a black ball inside. This opened box is taken off and you have a choice. You may confirm your initial guess that the white ball is in the box pointed by you from the very beginning or may change your opinion and choose the other box. What should be your choice?

A:

One chance per three that the white ball is in the box you have chosen initially. What about the other closed box? Does it have the same chance or not?

T:

This problem also requires the minimum knowledge of the probability theory. A common sense of course is rather helpful. Let give it 2 stars.

## 1.4. Mathematical problems



### 15. Purchase of a book\*

S:

Two girls, Kathy, 10 and Susan, 12 wanted to buy a book. It turned out that neither of them has enough money to buy the book and in fact, Kathy is short of 2 dollars and Susan of 3 dollars. They decided to join their money and to buy one book for two. However, even so they still did not have enough money for the book. How much did the book cost?

A:

To make the life easier let us consider only integers for this problem. So each girl has an integer amount of dollars and correspondingly the cost of the book is also integer.

T:

If we consider cents as well then the problem does not have a unique solution. But the reader may still determine the limits of the book price however. Let it be an additional training for one more star.

### 16. Scattering of stones\*\*\*

S:

There are  $n(n-1)/2$  stones which arbitrary divided into several separate groups. One chooses a stone from every group and combines them within a new group. Then this procedure is repeated again. Show that this procedure ends up with  $n-1$  groups such that the  $i$ -th group contains exactly  $i$  stones.

A:

Well, if I understood it right there could be groups that contain 1 or 2 or some other amount of stones. I even could imagine that it could be the only group that contains all stones. But if I take a stone from every group, the groups that contained 1 stone disappear. But instead a new group is created each time that combines one stone from every existed group. Also the group that contained  $k$  stones transforms to the group that now contains  $k-1$  stones. So it seems this process will never end up.

T:

Yes, it will. I give you a hint. Considering the process ended does not mean you stop to move the stones. In contrast you proceed but the final distribution does not change. In other words, suppose you had a certain amount of groups with 1 stone, some other amount of groups with 2 stones etc. It could be no groups that contained say  $k$  stones. So the process ends up when the number of these groups (or the distribution of stones over the groups) remains constant with time. This problem deserves 3 stars.

### 17. Problem of mixing\*\*

S:

I have a half-cup of tea and a half-cup of coffee. I take one teaspoon of tea and mix it with my coffee, then I take one teaspoon of this mixture and mix it with the tea. Which cup contains more of its original contents?

A:

Oh.., it is necessary to calculate percentages to figure it out. But I have feeling that this problem could be solved based on a common sense having not calculated anything.

T:

Yes, this problem can be considered as a pure math problem. Requires some knowledge of elementary math, not more. Good training and not that hard. But you are right, someone may guess the answer at once. Anyway, totally two stars.

### 18. Two cans\*

S:

How to measure 4 gallons if you have two cans - 3 and 5 gallons?

A:

Supposedly you have no more cans but on the other hand, you have water as much as you want. But the problem is where can you pour it in? But instead you may pour it out to any place.

T:

Not a difficult but funny problem. One star.

### 19. Problem of sons\*\*



S:

Two old friends John and Joe met at a party.

“Hi, Joe, How have you been?”

“Hi, John, Glad to see you, How are you?”

“I am fine, thank you. You know I have three sons.”

“Great! How old are they?”

“Well, if you multiply their ages you get 36”

“Let me think... No, I cannot find out their ages. You should give me more information.”

“O’key. The sum of their ages equals to the number of people in this room.”

“This is a bit better but still not enough.”

“Well, what if I tell you that my oldest son has brown eyes.”

“Oh, now I can figure out their ages easily.”

Can you?

A:

Hm...his situation is easier because he knows how many people are in the room. It looks hard for me to find an answer.

16

T:

This problem is not too hard. Actually, the statement contains enough information to find out the ages. Go ahead! And get your 2 stars!

### 20. Problem of doors<sup>\*\*</sup>

S:

A very long hallway has 1000 doors numbered 1 to 1000; all doors are initially closed. One by one, 1000 people go down the hall: the first person opens each door, the second person closes all doors with even numbers, the third person closes door 3, opens door 6, closes door 9, opens door 12, etc. That is, the  $n$ th person **changes** all doors whose numbers are divisible by  $n$ . After all 1000 people have gone down the hall, which doors are open and which are closed?

A:

Theoretically we can keep counting the state of each door, but how much time does it take! Wow! But I have an idea. I would rather write a program for my computer. The algorithm is pretty clear.

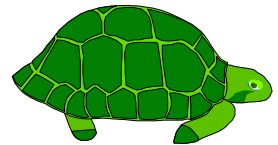
T:

You should better think about the problem. It is not hard and can be resolved theoretically in a few minutes. I would give it 2 stars.

### 21. Creeping turtles<sup>\*\*\*\*</sup>

S:

Four turtles are located each at a vertex of a square of side length 1m. Turtles start moving simultaneously with constant speed 1m per min. Their initial velocities are directed along the sides of the square counterclockwise. At any other moment of time each turtle moves strictly along the line that connect it with the neighboring turtle. Find the time when all four turtles meet.



A:

Hm...the time depends on the path each turtle passed before they met. It seems to be some kind of a spiral. But how one can calculate its length? Shall I apply the high level of math to resolve this problem?

T:

You may, of course. However, there exist a simple but not evident solution that does not require special math skills. So you better use your mother wit! And you will deserve 4 stars.

### 22. Defective coin<sup>\*\*\*</sup>

S:

Among 1000 coins one is defective: it has two heads. One chooses a coin (a good or bad one) and tosses it 10 times. It turns out that the head comes out all 10 times. What is the probability that the head comes out again when the coin is tossed one more time?



A:

I need to know whether we can treat all tosses as independent. Otherwise it is very difficult (at least for me) to find a probability of all these events.

T:

OK, consider all these tosses to be independent. If now you can find the solution – you deserve 3 stars.

23. What is bigger? \*\*\*\*\*

S:

Which of the numbers is bigger:  $e^\pi$  or  $\pi^e$  ?

A:

Sounds nice:  $e^\pi$ ,  $\pi^e$  ... It must be a trick to figure it out what is bigger.

T:

Maybe but I am not familiar with it. At this point you definitely need to use at least one idea borrowed from higher math. Yes, yes, it is necessary to refresh your math knowledge sometimes. Not easy - 5 stars.

## 1.5. Physics problems

24. 3 Lights \*\*\*\*\*



S:

You are in the first room of a two-room apartment. There are three individual switches each connected to a lamp, which are located in the second room. The rooms are separated by a door that is closed, so you cannot see the lamps. You are allowed to turn each switch on and off and enter the second room only once. Describe the procedure that allows you to determine which switch is connected to which lamp.

A:

Thus you may play with the switches in the first room as long as you want. Finally, you enter the other room to look at the lamps. The certain switches could be turned on, the others – turned off, and correspondingly some lamps could have light and the others could not. With two lamps and two switches everything would be so easy! But what to do with the third one?

T:

This problem requires a non-standard approach and a piece of common sense to be solved. 2 stars.

### 25. Train on the bridge\*

S:

A train passes over a bridge of 450 m in length within 45 seconds and passes past a telegraph-post within 15 seconds. What is the length of the train and its velocity?

A:

I think it is important to take into account the difference between the bridge and the post. I know a lot of differences that however are absolutely useless for solving this problem. I guess the only useful feature is that they have a different length.

T:

It is a pure math problem, not hard, but not so trivial as you can imagine. But nevertheless only 1 star.

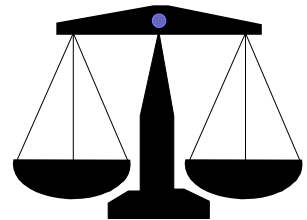
### 26. Communicating vessels on a balance\*\*\*

S:

Communicating vessels with water fixed attached to a balance. You put a pound of cork in one vessel and a pound of lead in the other vessel. What does it happen with the balance?

A:

It reminds me a physics class at school. Something like: each body being submerge in a water loses the weight equal to the weight of water having the volume displaced by the body...But I exactly remember how surprised I was having got this problem at my university exam on physics. Indeed, all people know that communicating vessels always have the equal level of liquid. And the weights of the cork and lead are also equal. But I felt that there is a certain trick...



T:

Yes, you are right. The Archimed law is applicable to this problem but not only this. Again add a piece of common sense and your physics intuition. I believe this problem deserves 3 stars.

### 27. Bird and cars\*



S:

Two bikers start toward each other having an initial distance 150 miles between them. At this moment a little bird also starts to fly from one of these biker to the other having a speed 15 mph while the bikers have a speed 12 and 13 mph respectively. Therefore it reaches the second biker in a certain point of his way, immediately turns back and flies toward the first biker etc until the bikers meet. How many miles does this bird pass till the bikers meet?

A:

Why did you place this problem among the physics ones. To me it is a pure mathematical problem. Certainly, mechanics says to us that the distance is a product of time and velocity. But if so, further I consider the distance covered by the bird before meeting the second biker, then...

T:

Stop, stop, easier! Wait a minute! Not all the problems require a direct calculation. You may spend a lot of time doing in such a formal way and make a mistake on the way. Therefore, calm down and think a bit more. This problem can be easily solved. 1 star.

## 1.6. Programming problems

### 28. Memory economy\*\*

S:

You have two integers A and B. You need to exchange them without having used any additional memory. In other words you need to store the value of B in the memory first occupied by A and respectively in the memory first occupied by B the value of A should be stored.

A:

Usually to exchange two objects, two integers in this particular case, we use some intermediate depository – buffer. Suppose A is stored in a part of memory 1 while B – in a part 2. We put the value of A into the buffer, then copy the value of B to 1 and finally copy the value of A from the buffer to 2. So we exchange these two integers by places. The idea of the problem is that you have no any buffer. May be we can use as the buffer some cells of memory just in the part 1 or 2?

T:

May be but only in some particular case. Generally you can not. But you can do it using a very unusual trick. Try to guess it! This idea deserves to get two stars.

### 29. Linked lists\*\*\*

S:

Given a linked list. Provide an efficient algorithm checking whether this linked list has a cycle.

A:

First of all what is a linked list? As I remember it is a set of structures of the same type. Among other elements each structure contains a pointer to the next structure. The last structure in the list contains the pointer referred to null. Linked list has a cycle when there is a structure with the pointer referred to any of the previous structures. Linked list does not have a cycle if there is a structure with the pointer referred to null.

T:

You explained it right. Now all you need is to find this algorithm. And to deserve 3 stars.

20

### 30. Binary representation\*\*

S:

Given a certain large positive number. You need to print its binary representation.

A:

Hm...it seems first we need to transform this number to the binary form and then try to print these binary digits as characters. But what about if the original number is very large. It could take a lot of time to make this transformation.

T:

Actually, this problem can be solved more elegant. Try to write a code and keep it as your own standard function. It could be of use for many applications.

### 31. Arrays\*\*\*

S:

Given array of integers  $x[1] \dots x[m+n]$  considered as a concatenation of two its parts: the beginning  $x[1] \dots x[m]$  of the length  $m$  and the end  $x[m+1] \dots x[m+n]$  of the length  $n$ . Exchange the beginning and the end not using any additional memory.

A:

It sounds very similar to the problem of memory economy. What about to use the same approach to solve this problem as well?

T:

It is not a bad idea. But, please bear in mind that a piece of the array is not a single integer as it was in the above mentioned problem. So this problem is more complex and respectively I assign 3 stars to it.

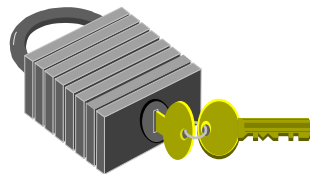
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# Solutions

## 3.1. Common sense



### 1. How to send brilliant\*\*\*\*

Surprisingly this problem has a rather simple solution however finding it is not so simple. The key point is that an ordinary locker locks all steel boxes used to send a parcel. Therefore, one can put MORE THAN ONE locker on the same box loop! Now the solution becomes clear. The first man sends a locked box with a brilliant inside to his friend to another city. This friend receives this box, PUT HIS OWN LOCKER just at the same loop and send this box locked with TWO lockers back. Then the first man receives this box and takes his own locker off. But the box is still closed therefore he surely can send it back again to the second man. Finally this man receives the box and takes the locker off because it is his OWN locker and he has a key for it. So the box is opened and he gets his brilliant.

### 2. Two robs\*\*

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If a rope being set on the fire from one side completely burns down within an hour, it completely burns down within half an hour being set on the fire simultaneously from both sides. It is always true even when the rope burn non-uniformly. Certainly in such a case two tongues of flame meet each other in a certain point out of the center of the rope. So we can measure 30 minutes. Now we need to know how to measure 15 minutes more. Based on the initial idea of setting a rope on the fire simultaneously from both sides we could solve the problem having another rope that burns down within 30 minutes. Fortunately, it is possible to realize both these ideas with given two ropes in use. Thus, we are doing as follows. First we set the first rope on the fire from both sides

and simultaneously set the OTHER ROPE on the fire from ONE side. When the first rope completely burns down (remember it happens exactly in 30 minutes) we set the OTHER SIDE OF THE SECOND ROPE on the fire. As this rope has been already burned within 30 minutes, it needs 30 minutes more in order to burn down. Therefore the rest unburned part of this rope will also burn 30 minutes. But at the moment we set the other side of the rope on the fire. So by analogy the tongues of flame will meet each other in 15 minutes. Totally it takes 30 minutes (the first rope) + 15 minutes (the rest part of the second rope) = 45 minutes.

### 3. Alphabetical series\*

This series *O T T F F S S E N* .....is the first letter of numbers 1, 2, 3, 4, ...in we read them in English, i.e. One, Two, Three, Four. Accordingly, the next letter has to be T (Ten).

### 4. Numerical series\*

In this triangle

1

11

21

1211

11221

312211

read the numbers in the following way: one 1 (makes 11),two 1's (makes 21), one 2 one 1 (makes 1211), and so. Accordingly, the next row is 3112221.

### 5. Problem of crossing\*

Two people come to the riverside and see an empty boat. But it may transfer only one guy. So how they both can cross the river? They can if they come to the DIFFERENT sides of this river!. Since it was not stated in this problem to which side of the river those two people have come let consider them having come to the different sides! Thus one of them takes the boat (the one on which side the boat originally was) and crosses the river. On the other side of the river he passes the boat to the second person and he/she cross the river in the same way. So the solution is just funny and needs only a non-standard mind to be found.



## 3.2. Logic problems

### 6. Town of liars\*\*



Analyzing the problem we see that it is necessary to find a right direction while the comer says only Yes and No. Therefore you must point to a certain road in your question. Possible replies depend upon whom do you meet – a

liar or not. But in order to solve a problem they both should provide the same answer. Otherwise it is impossible to figure out where which city is. Suppose we prefer the following scheme of replies presented below as a table

Town you are pointed to	You meet	
	Truthful man	Liar
Of truthful people	<b>Yes</b>	<b>Yes</b>
Of liars	<b>No</b>	<b>No</b>

So you have to ask a question that could be answered according to this table. What should be the question on which both the liar and truthful person give the same answer? This question probably has to touch upon the person you are talking with. For instance, it could be the following question. You point to the town and ask the comer: “Do you live in this town?”. Indeed, if it is the liar’s town the liar will say “NO” and the truthful man will say “NO” as well. But if it is a town of truthful people its citizen will answered your question “YES” while a liar will say “YES” also because he lies. Thus, reply “NO” means that you pointed to the liar’s town while reply “YES” means that this one is the town of honest people.

### 7. Four sages\*\*

Every sage saw that the old man has totally 5 caps while only one of them is white. In addition every sage sees that all other 3 sages are worn by a black cap. Accordingly, each sage can imagine that equiprobably he has either black or white cap on his head. But the cleverest sage thought further: “Suppose I am worn by a white cap”. It means that all other sages see it. But as the white cap is unique they would immediately realized that they are worn by BLACK caps. But they KEEP SILENCE! It means that I have a BLACK cap either!”.



### 8. Five sages\*\*\*

Every sage saw that the old man has totally 7 caps while 2 of them are white. Also every sage sees that all other 4 sages are worn by a black cap. Accordingly, each sage can imagine that his own cap might be either black or white because the rest 3 caps (7caps – 4 sages with caps on the head) are: 2 white and 1 black. Again the cleverest sage could treat the situation as follows. “Suppose I have a white cap. Then all other sages see 3 black and one white (mine) caps. As there exist the other white and black caps each of these sages do not know what cap he is worn. But let one of them (sage 2) suppose that he has the white cap. So he could conclude that white caps cover him and me while the other sages see it. Therefore, they would manage to surmise that they are worn with black caps but they did not. Therefore, sage 2 considers himself to be worn with a black cap. But he could say about it while he is quiet. That is why I fail having supposed that I have the white cap and so I have the black one!”

### 9. Lost dollar\*\*

The final question is stated so that it makes this problem knotty while it is solved quite easy. Three men paid \$27.00 and this money has been distributed between the waitress (\$25 = \$30 (initial payment) - \$5 (change)) and the boy (\$2). That is it! This problem requires certain common sense to realize that \$2 must not be added to the original \$27 but instead should be subtracted!



### 10. Honest people\*\*

There are 8 honest people in the room.

Indeed, suppose there are  $k$  honest people in the room. In such a case the first guy is a liar because he convinces us that there are no honest people at all. Similarly all guys who assert that there are no more than 1, 2, 3... $k-1$  honest people in the room are liars. Thus,  $k$  first people lie and therefore the rest  $16-k$  tell truth. On the other hand we assumed from the very beginning that there are  $k$  honest people in the room. So we arrive to the equality  $k = 16 - k$  from what one easy obtain  $k = 8$ .

### 11. Town of unfaithful wives\*\*\*



It turns out that this problem is similar to the problem on sages and black and white caps. By analogy suppose that there is the only adulteress in the city. Then her husband immediately will know about it because i) he knows everything about all the other wives so he knows that they are faithful, and ii) he knows from the pilgrim that there are adulteresses in the town. That is why soon he brings his wife to the court

Now let us assume that there are exactly 2 unfaithful wives in the town and number them as 1 and 2. As far as every husband has an information about all the other wives except his own one, the husband of woman 1 knows that woman 2 is unfaithful while he does not know anything about his wife and similarly the husband of woman 2 knows that woman 1 is unfaithful. But for instance, the husband of woman 1 could think in the following way. "Suppose my wife is faithful. Therefore there is the only adulteress in the town. But according to the previous consideration her husband would guess it immediately and bring her to the court. But he did not do this while the first day was over! It means he is not able to determine whether his wife is faithful or not. Why? Apparently, because he is aware of at least one more adulteress. But I know exactly everything about all women in the town except only my wife and I know that only woman 2 is unfaithful. Thus, another unfaithful woman is my wife!"

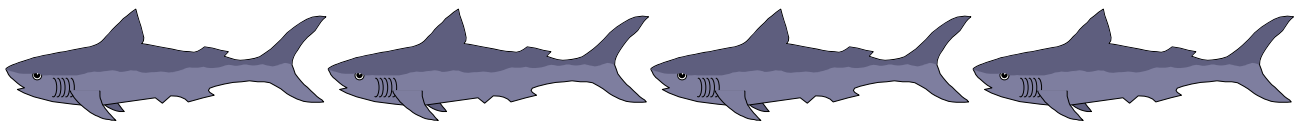
What is important that he could make this conclusion only on the second day after the pilgrim came to the town because the first day he is waiting for the activity of husband 2 (remember that husband 1 knows that woman 2 is unfaithful). Thus, he brings his wife to the court on the second day. As husband 2 argues in a similar way he also brings his wife to the court same day.

Further, by analogy if there exist  $n$  adulteresses in the town their husband knows about the  $n-1$  ones while all the other husbands are aware of all  $n$  adulteresses. If within  $n-1$  days nobody brings his wife to the court so for the  $n$ -th day all  $n$  husbands brings their unfaithful wives to the court. Thus, revealing of all adulteresses requires as many days as the number of adulteresses.

For readers with a mathematical background note that this conclusion can be proved using the method of induction.

## 3.3. Search of strategy

### 12. Herrings in a barrel\*\*





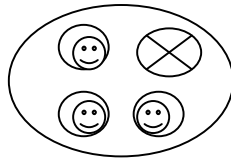
First, as we have to put both hands to the barrel at once we may do it in two ways, namely as it is presented in the pictures. Here a black circle denotes a hole to which we put a hand while a white hole is free. Thus, our hands can be put either diagonally or horizontally (or vertically that is the same). As the barrel is rotated no matter which diagonal or which horizontal or vertical line is actually chosen.



Then it is easy to get 3 herrings oriented in the same direction. Indeed, first we might, for instance, put the hands to the diagonal holes, take the herrings and set them, say “face up”. After rotating the barrel we put now our hands to the horizontal holes. Surely one of this holes contains a herring set “face up” because either the first hole or the second one belong to the diagonal where we already set herrings “face up”. Therefore we may set these two “horizontal” herrings “face up” again and thus at least 3 herrings are already set “face up”. But it may happen that after next rotations you never choose the hole with the last herring oriented “face down”. So what should be the right strategy in this case?

Remember we have a situation where 3 herrings are oriented “face up”. Suppose that after the next check (after the next rotation) we have found 2 herrings “face up” (1 and 2, see a picture below). Otherwise, if the hearing are oriented in opposite directions, say “face up” and “face down” we simple turn the herring with “face down” downside-up and the barrel will be opened!

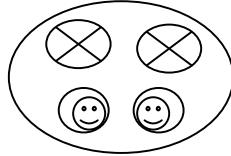
1



2

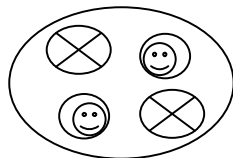
*Step 2*

If we turn them both over we arrive to the previous situation with the only difference that now 3 herrings are situated “face down” that certainly make no sense. Therefore we turn one of them over in order to set it “face down”. It means that we arrive to the following picture



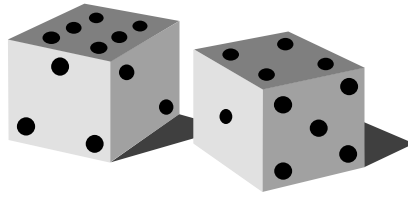
*Step 3*

Now after the next rotation we have to check to horizontal holes! It is obvious that if these taken herrings are oriented in the same direction we simply turn them both over and the barrel will be opened. Otherwise, if they are differently oriented we turn EACH of them over that yields



*Step 4*

The last step after the next rotation is to put the hand in diagonal holes and turn both herrings back. After that the barrel will be opened. Counting all the steps gives the solution of the problem, namely: following the described strategy the barrel could be opened maximum at 5 steps.

**13. Dices\*\***

Solving of the problem requires a reader to have some initial knowledge of the probability theory. We have already mentioned that in case of two possible attempts your strategy is quite obvious, namely: it would be reasonable to roll the dices again if the probability of getting more points after the second attempt is more than 0.5. Thus, if you get 1 at the first attempt the probability to get more (2,3,4,5 or 6) at the second one is  $5/6 > 1/2$ . Accordingly, the probability to get more than 2 is  $4/6$ , more than 3 –  $1/2$ , more than 4 –  $2/6$  etc. Thus, if you get 4,5 or 6 after the first roll – take your money. Otherwise (1,2 or 3) it is reasonable to roll the dice again.

A similar scheme has to work in case of 3 attempts as well. We should roll the dice again after the first attempt if the probability to obtain more points within the next two attempts is more than  $1/2$ . Thus, if 6 comes at the first roll you surely have to take money. If it is 5, the probability to get 6 within the next two attempts is  $1/6 + 1/6 = 1/3 < 1/2$ . It means this is the case you have also to take money. If you get 4 after the first attempt, the probability to get 5 or 6 within the next two attempts is  $4(1/6) = 2/3 > 1/2$ . Thus, it is reasonable to roll the dice again. The same is true for 1,2 or 3 coming after the first roll.

**14. Choice of balls\*\***

At first sight the arisen quiz sounds strange because you have to choose one of the boxes anyway. But let us compare your chances. Suppose you follow your initial choice. The probability of getting a white ball in this box is  $1/3$ . But what is the probability to find the white ball in another box that has not been opened. After you made a choice and marked a box where you expect to find the white ball the probability to find the white ball in 2 left boxes is  $2/3$ . But then you may see that there is no the white ball in the opened box. It means that the probability  $2/3$  is now related to the third box. It is twice more than the probability to find the white ball in first box, thus it is reasonable to change your initial choice and open the third box.

For disbelievers: consider the situation where there are 100 boxes instead of 3. After you have chosen one, someone opens 98 of the remaining 99 boxes that do not contain the white ball. Should you change your choice with the last remaining box? Surely yes, because the probability is very large ( $99/100$ ), i.e. 99 times more than for the first box!

## 3.4. Mathematical problems

**15. Purchase of a book\***

Suppose the book cost  $X$  dollars. So Kathy has  $X-2$  dollars while Susan –  $X-3$ . After they joined their money together they have  $2X-5$  dollars but it is still not enough, i.e.

$$2X - 5 < X$$

or  $X < 5$ . On the other hand Kathy had some money as well as Susan, i.e. each girl had more than zero. It yields two additional inequalities

$$X - 2 > 0$$

$$X - 3 > 0$$

Analyzing all these inequalities we arrive to the following one

$$3 < X < 5$$

Thus, the cost of the book is 4 dollars while Kathy had only 2 dollars and Susan had 1 dollar.

Another solution. If Kathy is short of only 2 dollars for buying a book and if the joint money is also not enough to purchase it, Susan can not have more than 2 dollars. So she has one. But since she is short of 3 dollars, the book costs \$4.

## 16. Scattering of stones

According to the statement of the problem some groups disappear with time while other new groups appear. The only possibility to end the process up is to arrive to a certain steady distribution of stones such that after any next rearrangement of the stones all the groups would be kept. It does not mean that each group does not change with time because as we know we permanently have to take one stone from every group and collect them within a new one. But it means that after the end of this step the number of groups with one stone, two stones etc remains constant.

When rearranging each step one decreases the number of stones within a certain group by one until it completely disappears. Let us consider a steady state. After the next rearrangement group 1 that contains exactly 1 stone disappears. But we consider the steady state therefore instead of this group another one should be produced that again contains exactly one stone. Such a group can be produced only from a group 2 that contains exactly 2 stones. Thus at least one group 2 must be presented in the steady state. But it means that at least one group 3 that contains exactly 3 stones must be presented in the steady state, otherwise we have no chance to get group 2 after the next rearrangement. Consequently we discover that groups 4, 5, 6 etc must be presented in the steady state. So where do we have to stop? A simple analysis shows that the last group must be group  $n-1$  that contains exactly  $n-1$  stones. Indeed, if we have one group 1, one group 2 ... one group  $n-1$  so after one stone is taken from every group, group 1 disappears, group 2 becomes group 1, group 3 becomes group 2 ... group  $n-1$  becomes group  $n-2$ . At the same time all taken stones, namely 1 from group 1, 1 from group 2 ... 1 from group  $n-1$  produce new group  $n-1$ ! Thus, we returned to the previous configuration that means this is indeed steady state.

The last verification implies that we have to count the total number of stones within all groups. It is easy to see that the numbers of stones over all groups form an arithmetic progression. As known from elementary mathematics the sum of all its numbers is  $n(n-1)/2$  that just coincides with the total initial number of stones. Therefore, we proved that our solution is consistent with the statement of the problem.

## 17. Problem of mixing\*\*

Suppose each cup contains a unity (conditional) of liquid while each teaspoon contains  $x$  portion of liquid. Certainly,  $x < 1$ . After one teaspoon of tea is taken from the first cup, it contains  $1 - x$  tea measured in the conditional units. When you mixed tea from this teaspoon with coffee the second cup will contain  $1/(1+x)$  units of coffee and  $x/(1+x)$  units of tea. Then you take a teaspoon from the second cup. This teaspoon contains  $x/(1+x)$  units of coffee and  $x^2/(1+x)$  units of tea. Accordingly, the second cup now contains  $1/(1+x) - x/(1+x) = (1-x)/(1+x)$  units of coffee. After you added the content of the teaspoon to the first cup it will contain  $1 - x + x^2/(1+x)$  units of tea. It easy to check that

$$1 - x + \frac{x^2}{1+x} = \frac{1}{1+x} > \frac{1-x}{1+x}$$

Thus, tea in the first cup will be more than coffee in the second cup.

### 18. Two cans\*

It is clear that to solve this problem one has to carry out a few pourings from one cup to the other one. For instance, it could be as follows.

- Put 5 gallons of water into the 5-gallons' cup;
- Pour 3 gallons from this cup to the other one. Thus, in the first cup 2 gallons of water left;
- From the 3-gallons' cup pour all the water out and instead pour into it all water (2 gallons) from the 5-gallons' cup.
- Again fill the 5-gallons' cup full of water;
- Pour water from the 5-gallons' cup into the 3-gallons' cup until the last one is full. As this cup already contained 2 gallons actually we poured exactly 1 gallon from the 5-gallons' cup;
- Respectively, in the 5-gallons' cup now we have  $5 - 1 = 4$  gallons.

### 19. Problem of sons\*\*



Here are all possible variants how to present 36 as a product of three factors

- |            |           |
|------------|-----------|
| • 1, 1, 36 | • 1, 6, 6 |
| • 1, 2, 18 | • 2, 2, 9 |
| • 1, 3, 12 | • 2, 3, 6 |
| • 1, 4, 9  | • 3, 3, 4 |

Certainly, as there exist 8 possible combinations John being aware of only the product of sons' ages – 36 - can not determine their ages. On the other hand he knows how many guests are at the party. Therefore if he is still not able to figure out the ages of Joe's sons, there should be two combinations that have the same sum. They are: 1, 6, 6 and 2, 2, 9. But when Joe mentions his oldest son, that rules out the possibility of 1, 6, 6 combination because it does not contain the only oldest son. Thus, the solution is 2, 2 and 9 years.

### 20. Problem of doors\*\*

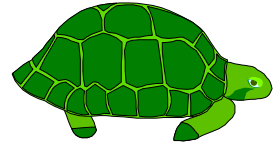
Obviously, the status of each door is changed so many times how many divisors the number of the door has. For instance, all prime numbers are divided only by unity and itself. Therefore, for a door with the prime number  $k$  the first person opens the door while the  $k$ -th person closes it. By analogy, only those doors will be opened which number has the odd amount of divisors.

Let us analyze the situation. Each number  $k$  has at least a pair of divisors – unity and itself. Suppose, in addition this number has another divider, say  $n$ . Then it could be represented as a product  $k = n * l$  where  $l$  is a quotient of dividing  $k$  by  $n$ . However, it means that  $k$  is also divided by  $l$ . Thus, for each additional divisor of  $k$  there is a paired divisor such that the product of these divisors gives  $k$ . In turn it means that the number of all divisors has to be even.

Is there any exception. Yes, it is. If number  $k$  is a square of the certain number  $l$ , so again it could be represented as a product  $k = l * l$ , but in this case we have only one divisor, because the quotient equals the divisor! Thus, the only situation may happen that some number  $k$  has the odd number of divisors when this number  $k$  is a square. Accordingly, all doors with numbers 1, 4, 9, 16, 25... will be opened while all the others will be closed.

### 21. Creeping turtles

Apparently, the simplest way to solve this problem is to consider the movement of all turtles being in the coordinate system rigidly connected with one of the turtles, for instance with turtle 1. In this system turtle 1 does not move while according to the statement of the problem at any moment of time turtle 2 goes exactly along the shortest way between it and turtle 1. It means that despite in the new coordinate system the surface of the land performs a very complex movement, the shortest way between these 2 turtles is a straight line. It exactly coincides with a side of the initial square where all turtles have been located at the initial moment of time. As the length of this line does not depend on the coordinate system chosen, it is 1 m while the velocity of the turtle is 1m/min. Therefore, turtles 1 and 2 will meet in 1 min. But this time does not depend on in what coordinate system we consider this movement!



In view of symmetry of the problem (we could arbitrarily choose any turtle) in the original coordinate system connected with the land all turtles will meet in 1 min in the center of the square. The path of each of them is a complex curve (like a spiral) but its length is exactly 1 m.

### 22. Defective coin

The probability to choose a defective coin among 1000 coins is 1/1000. The probability that the defective coin comes out by the head is 1. Thus, the probability to get the head successively coming out 10 times is  $0.001 * 1 * 1 \dots * 1 = 0.001$ .

The probability to choose a normal coin among 1000 coins is 999/1000. The probability that the normal coin comes out by its head is  $\frac{1}{2}$ . If we assume all tosses to be independent, the probability to get the head successively coming 10 times is  $0.999 * (\frac{1}{2}) * (\frac{1}{2}) \dots * (\frac{1}{2}) = 0.999 / 2^{10} = 0.001 * (999 / 2^{10})$ .

Thus, the probability that we tossed the defective coin is  $a = 2^{10} / 999$  times more than the probability that it was a normal coin. In other words, this coin is defective with probability  $p_d = a / (1 + a)$  while it is normal with probability  $p_n = 1 / (1 + a)$ . As the probability to get the defective coin by "head up" is 1 and the probability to get a normal one by "head up" is  $\frac{1}{2}$  the final probability that the coin comes out by its head is  $1 * p_d + \frac{1}{2} * p_n = 3047 / 4046 \approx 0.75$ .

### 23. What is bigger?

Let us present  $\ln \pi$  as

$$\ln \pi = 1 + \ln \left[ 1 + \left( \frac{\pi}{e} - 1 \right) \right]$$

As  $\pi$  is a bit more than  $e$ , the ratio  $\pi/e$  is a bit more than 1, therefore  $0 < \frac{\pi}{e} - 1 < 1$ . Further we use an inequality

$$\ln(1 + x) < x$$

at  $0 < x < 1$  (see the plot below).

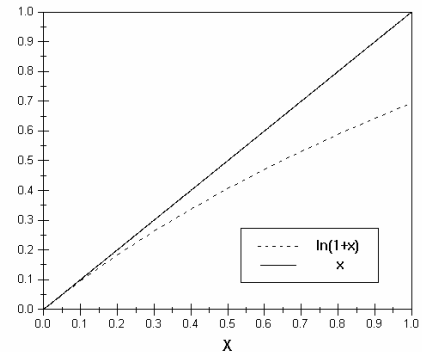
Those who is familiar with mathematical analysis can easily prove this inequality expanding  $e^x$  into the series -

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}, \text{ therefore } e^x > 1 + x \text{ or } \ln(1+x) < x.$$

From these two expressions we find

$$\ln \pi < 1 + \frac{\pi}{e} - 1 = \frac{\pi}{e}$$

$$e \ln \pi < \pi$$



and finally  $\pi^e < e^\pi$ .

## 3.5. Physics problems



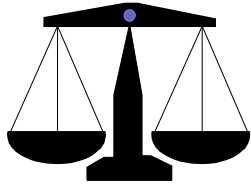
### 24. 3 Lights\*\*\*\*

Obviously there is no sense to turn on all 3 switches. But if we turn on 2 of them we will only manage to find out that the switch off duty corresponds to the lamp out of light. The same is true if we turn on only one switch because this switch will corresponds to the lamp on the light. Therefore, it seems the problem can not be solved. However, it can. The idea is that nobody said that we might only look at the lights. Indeed, for instance we can touch them. The matter is that if this lamp was on the light for a long time but then was turned out, it continues to be HOT for a certain time in contrast to the lamp that has not been on the light at all! From here we get the following solution. First you have to turn on one of the switches (say switch 1) and wait for a certain time (say few minutes). Then you have to turn this switch off and turn on another one (switch 2). After that you enter the room and see what lamp is on the light. It is clear that it corresponds to switch 2. Then you touch the other lamps. Which of them is warm that corresponds to switch 1 while the last one – to the residual switch..

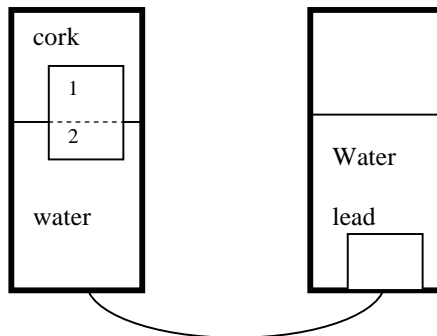
### 25. Train on the bridge\*\*

15 seconds the train moves past a telegraph-post. It means that same 15 second it needs to drive down the bridge. Thus, the interval of time from the moment when the train comes into the bridge till the moment when it starts to ride out the bridge is 30 seconds. So within 30 seconds the train passed over 450 meters whereas its velocity is 900 meters per minute or 32.7 mph. The train passes the path equal to its length during 15 seconds. Therefore, its length is  $900 \cdot \frac{1}{4} = 225$  meters.

## 26. Communicating vessels\*\*\*



The change of balance (if ever) could be associated only with the different behavior of two substances put into the vessels. The cork will swim in the water while lead will go to the bottom. As the vessels are communicating the level of water in both vessels is the same at any time. Thus, we may depict this situation as shown in the picture.



According to the Archimedean law the weight of the cork is equal to the weight of water that would occupy volume 2 in the picture (in the picture the cork consists of two parts: 1 – the above water part, and 2 – the underwater part). It means that we could remove the cork and instead put the water in the volume 2. After that we have two same vessels with the same level of water. The only difference is that the right vessel there is lead while in the left vessel the same volume at the SAME PLACE is occupied by water. As lead is heavier than water the balance will bend to the right.

## 27. Bird and cars\*



Because to find this distance directly by counting the length of all flies is too cumbersome the idea of the solution is to determine a time necessary for bikers to meet each other. It is easy to see that they approach each other with the total velocity 25 mph therefore they will meet in 6 hours. All this time the bird is flying with a speed 15 mph. Thus within this time it will cover a distance of 90 miles.

# Programming problems

## 28. Memory economy\*\*

The first solution is appropriate for any programming language. You perform three operations:

$$A = A + B$$

$$B = A - B$$

$$A = A - B$$

Indeed, after the first operation  $A = A_0 + B_0$  where we denoted  $A_0$  and  $B_0$  the original values of A and B. After the second operation one has  $B = A - B = A_0 + B_0 - B_0 = A_0$ , i.e. B took the initial value of A. The third operation yields  $A = A - B = A_0 + B_0 - A_0 = B_0$ .

Another solution could be used in C with the help of the operation  $\wedge$  (XOR).

$$A \wedge= B$$

$$B \wedge= A$$

$$A \wedge= B$$

Indeed,

$$A \wedge= B = A_0 \wedge B_0;$$

$$B \wedge= A = A \wedge B = (A_0 \wedge B_0) \wedge B_0 = A_0 \wedge (B_0 \wedge B_0) = A_0 \wedge 0 = A_0$$

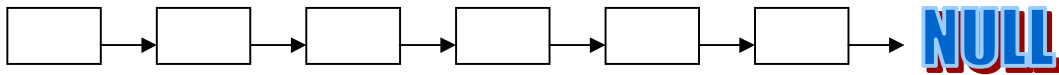
$$A \wedge= B = A \wedge B = (A_0 \wedge B_0) \wedge A_0 = (A_0 \wedge A_0) \wedge B_0 = 0 \wedge B_0 = B_0$$

You may also choose two arbitrary integers and just check that it is true.

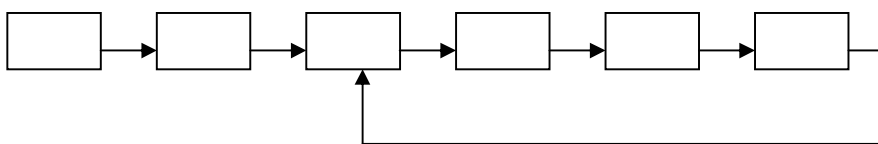
### 29. Linked lists\*\*\*

This problem can be solved in many different ways. We describe some of them. Certainly the reader can guess some new efficient algorithms unknown to the authors. If so we would appreciate him/her for getting us familiar with.

Let us remind again that linked list is a set of structures of the same type. Among other elements each structure contains a pointer to the next structure. The last structure in the list contains the pointer referred to null. Linked list has a cycle when there is a structure with the pointer referred to any of the previous structures. Linked list does not have a cycle if there is a structure with the pointer referred to null (see the Figures below).



**Fig.A No cycle**



**Fig.B Cycle**

The first idea how to check whether the linked list has a cycle or not is to mark structures where we have already been. For instance, let us consider linked list where each structure has the following form

```
typedef structure element {
    int          inform;
    structure element *next;
} data;
```



Here the initial value of “inform” is zero for all structures. When coming to each structure first we check the value of “inform”. If it is zero it means we have not been here. Thus, to mark this structure we change “inform” for instance, to be equal to 1. If we come to the structure with “inform” already equal to 1 it means the linked list has a cycle. Certainly, one can imagine many different ways how to mark the structure.

This algorithm can be schematically presented as follows:

```

element *p = <the beginning of the list>
while (p->next) {
    if (p->inform == 1) break;
    p->inform = 1;
    p = p->next;
}
if(p->next == NULL) <no cycle>;
else <there is a cycle>;

```

What is reasonable to do if there is no opportunity to mark each structure? Another idea is to reverse left arrow (see Fig. A) while we passed the structure. It can be done using 3 pointers and one additional passageway over that part of the linked list we have already passed. As it is easy to see if the linked list has the cycle this algorithm returns us to the first (initial) structure. The scheme is the following.

```

element *q,*p,*k = <the beginning of the list>
if (!(p=k->next)) <the list consists of one element>; exit(1);
q=p;
while (p != <the beginning of the list> && p->next) {
    q = p->next;
    p->next = k;
    k = p;
    p = q;
}
if(p == <the beginning of the list>) <cycle>;

```

A disadvantage of this algorithm is that we change the structure of the list.

Let us describe one more algorithm that seems to be very efficient. Instead of one pointer in each structure we will consider two pointers. The first one is a usual pointer that points to the next structure in the list. The other one is a pointer that runs twice faster. It means that when we are in the  $k$ -th structure this second pointer points to the  $2k$ -th structure. The idea is that if the list contains a cycle, it must happen that the second pointer overtakes the first one. Indeed, the existence of the cycle means that beginning from a certain time both pointers will come inside the cycle. Under their movement over the cycle the distance between the “slow” pointer and the “fast” one after each step will decrease by one element because the “fast” pointer “jumps” for two elements while a “slow” pointer – only for one. As the number of elements in the cycle is countable so the fast pointer overtakes the slow one at some time. Therefore, if the value of these two pointers becomes equal at some step it means that the list contains the cycle. A scheme of this algorithm is as follows.

```

element *p = <the beginning of the list>
if (!(p->next)) <the list consists of one element>; exit(1);
q=p->next;
while( (q->next != NULL) && (p != q) ) {
    q = q->next;
    if(!(q->next)) break;
    q = q->next;
    p = p->next;
}

```

```
if(q->next) <no cycle>;
   else      <cycle>;
```

### 30. Binary representation\*\*

For instance, it could be the following C code

```
#include<stdio.h>
#include<stdlib.h>
#include<string.h>
#include<math.h>

/* Function which returns a pointer to string with a binary
   representation of the given integer */

char *bytepr(int);

void main()
{
    int number;
    char *buf=NULL;

    puts("Enter some number");

AGAIN:
    scanf("%d",&number);
    if(number < 0) {
        printf("You entered a negative integer, try again\n");
        goto AGAIN;
    }
    else {
        buf = bytepr(number);
        printf("\ninitial number = %d\t binary = %s \n", number,buf);
        free(buf);
    }
}

char *bytepr(int num)
{
    int nhex, remind;

/* This is a pointer to string where we store a binary representation
   of the given integer num
   */
    char *bin;

/* Binary representation of hexadecimal numbers
   */
    char *hex[] = {
        "0000","0001","0010","0011",
        "0100","0101","0110","0111",
        "1000","1001","1010","1011",
```

```

        "1100", "1101", "1110", "1111"};

    if(!num) return(hex[0]);

    /* Count of how many hexadecimal numbers consists our integer num
    */
    for(nhex=0, remind=num; remind; nhex++) remind /= 16;

    /* allocate memory for the resulting string and fill it by zero
    */
    if ((bin = (char *)malloc(nhex*4*sizeof(char))) == NULL) {
        printf("Out of memory\n");
        exit(-1);
    }
    memset(bin,0,nhex*4);

    /* Calculate hexadecimal representation of num from left to right and
    substitute each hexadecimal digit by its binary representation
    */
    for(nhex=nhex-1; nhex; nhex--) {
        remind=num/pow(16,nhex);
        strcat(bin,hex[remind]);
        num -= remind*pow(16,nhex);
    }
    strcat(bin,hex[num]);
    return (bin);
}

```

Another version:

```

#include<stdio.h>
#include<stdlib.h>
#include<string.h>

void main()
{
    int number,k, remind,i;
    char *buf,a;

    puts("\nEnter some number");

```

AGAIN:

```

    scanf("%d",&number);
    if(number < 0) {
        printf("You entered a negative integer, try again\n");
        goto AGAIN;
    }
    else {
        /* How much space do we need to allocate for this binary?
        * we calculate the number of decimal signs K and then determine
        * the number of binary signs as  $L = (K+1)/\lg 2 = (K+1)/0.3$ 
        * that approximately is  $4(K+1)$ 

```

```

*/
for (k=0, remind=number; remind; k++) remind /= 10;
if ( (buf = (char *)malloc(4*(k+1)*sizeof(char))) == NULL) {
    printf("Out of memory\n");
    exit(-1);
}
memset(buf,0,4*(k+1));
printf("\ninitial number = %d\t binary = ",number);

/* find binary signs as a reminder of sequential dividing our
number by 2, but we store them in a reverse order
*/

for(k=0; number; k++) {
    *(buf+k) = number % 2;
    number >>= 1;
}

/* reverse the string to get binary signs in a right order
*/

for(i=k; i; i--) printf("%d",*(buf+i-1));
free(buf);
}
}

```

### 31. Arrays \*\*\*

This problem has been formulated in the book of Greece. Its solution consists of two parts.

- First you have to guess how to rearrange elements of the array having not used any additional memory. It means that if from the very beginning elements of this array were  $a[0], a[1], \dots, a[n]$  after this transformation they have to be  $a[n], \dots, a[1], a[0]$ . But we already considered a similar problem in the quiz “Memory economy”. Therefore, using that method first we exchange 1 and  $n$ -th elements of the array, then 2 and  $(n-2)$ -th etc.
- Now using this our knowledge how to rearrange the array elements having not used any additional memory we can exchange the beginning and the end of the concatenated arrays in 3 steps, namely. First we turn over the beginning, then we turn over the end. Finally we turn over the whole array which consists now of the inverted beginning and the inverted end. It is easy to check that this procedure gives rise to the solution of the problem.